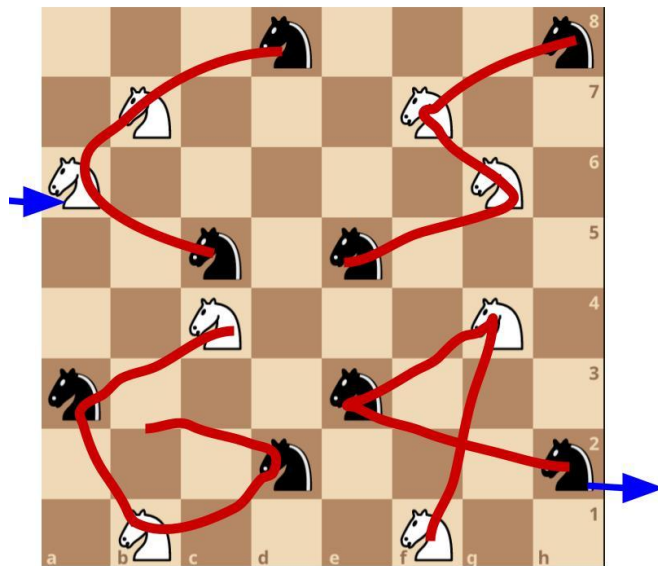


# Pre-class pondering

- Is it possible for a Sudoku to be uniquely solvable even if not all of the 9 numbers show up at least once?
- How few givens do you think a Sudoku can possibly have while remaining uniquely solvable? (This one can't be just reasoned out, AFAIK, but what would your guess be?)
- How many givens can a Sudoku possibly have while **not** being uniquely solvable?

# CS64: Computation for Puzzles and Games



**Autumn 2022**

**Lecture 3: Nikoli Puzzles, Part 1**

# Announcements

- For some reason Axess listed a final for this class. There isn't one, of course, and that's gone now.
- Please answer the course survey if you didn't last week. I will actually finally read them this week :\ my own schedule was too packed and I dropped CS111
- Speaking of: add/drop deadline is this Friday, 5 PM. Unfortunately there are no larger rooms available (I asked...)
  - If you end up still waitlisted, you can still show up and do everything you want
  - If you don't really need the unit, consider auditing instead

# Niemann Cheating Scandal Spirals Out Of Control As Magnus Carlsen's Rook Found Dead

| 9/23/22 6:45AM | Alerts



# Niemann Cheating Scandal Spirals Out Of Control As Magnus Carlsen's Rook Found Dead

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Chess.com also noted "anomalies" in Niemann's rate of improvement, which has seen him soar up the rankings in classical chess from around 800 in the world to the top 50 in less than two years.

The site said this rise was the fastest in "modern recorded history" and had occurred "much later in life than his peers".

But it found no evidence he had cheated in his game against Carlsen or in any other over-the-board games.

A separate statistical analysis of Niemann's over-the-board games by Prof Kenneth Regan, widely regarded as the world's leading expert on cheating in chess, has also found no evidence he cheated.

# Pre-class pondering

- Is it possible for a Sudoku to be uniquely solvable even if not all of the 9 numbers show up at least once?
  - Yes! Imagine taking a solved grid and deleting all the 9s, for instance. Doesn't work with two numbers, though
- How few givens do you think a Sudoku can possibly have while remaining uniquely solvable? (This one can't be just reasoned out, AFAIK, but what would your guess be?)
  - 17, amazingly. This has been proven to be tight.
- How many givens can a Sudoku possibly have while not being uniquely solvable?
  - 77. We'll see this a bit later

# Logic puzzles pre-Nikoli

	Jam	Marmalade	Honey	Marmite	21	18	15	12	Blue	Green	Yellow	Red
Peter						X						
Jane						X				X		
Simon					X	X	X					
Alice						X						
Marmite												
Honey												
Marmalade												
Jam												
12												
15												
18												
21												

(example from Wikipedia)

Clues like "The person who likes honey is older than Jane, who wears green"

# Logic puzzles pre-Nikoli

	Red	Yellow	Green	Blue	12	15	18	21	Marmite	Honey	Marmalade	Jam
Peter						X			X			
Jane			X			X			X			
Simon					X	●	X	X	X			
Alice						X			X			
Marmite	X	X	X	X	X	X	X	X				
Honey												
Marmalade												
Jam												
12												
15												
18												
21												

fact: nobody likes Marmite







# Sudoku is older than most think

- The Sudoku-esque puzzle to the right is from 1895! No boxes, but a diagonal constraint

eut la tête prise et écrasée par les tampons.  
La mort a été instantanée.

LÉOPOLD LEPREZ.

## DIVERTISSEMENTS QUOTIDIENS

N° 3879 — CARRÉ MAGIQUE DIABOLIQUE

Par M. B. Meyniel

Compléter le carré ci-dessous en employant les neuf premiers nombres chacun neuf fois de manière que les horizontales, les verticales et les deux grandes diagonales donnent toujours à l'addition le même total.

7	8	9	1	2	3	4	5	6
3				4				8
5				9				1
8				3				4
1	2	3	4	5	6	7	8	9
6				7				2
9				1				5
2				6				7
4	5	6	7	8	9	1	2	3

Ce carré devra être diabolique, c'est-à-dire que le carré restera magique si l'on place une ligne horizontale ou une colonne verticale à la suite de toutes les autres.

# Sudoku is older than most think

- The Sudoku-esque puzzle to the right is from 1895! No boxes, but a diagonal constraint
- Actual Sudoku were around in US logic puzzle magazines in the 80s as "Number Place"
- Pitched as Sudoku (数字は独身に限る, "digits must be solitary") in Nikoli in the 80s
- Blew up worldwide in the 2000s, misspelled ever since

But, if the use of numerals is optional, there is still plenty of mathematics to be found. In solving a sudoku puzzle, we consider the myriad ways of placing symbols in a grid. This involves the permutations of the symbols, a branch of mathematics known as Combinatorics. Mathematicians are keenly interested in a version of sudoku puzzle, where each row and each column must contain all symbols, but there is no

eut la tête prise et écrasée par les tampons.  
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3				4				8
5				9				1
8				3				4
1	2	3	4	5	6	7	8	9
6				7				2
9				1				5
2				6				7
4	5	6	7	8	9	1	2	3

Ce carré devra être diabolique, c'est-à-dire que le carré restera magique si l'on place une ligne horizontale ou une colonne verticale à la suite de toutes les autres.

# Latin squares

- An  $n \times n$  Latin square uses  $n$  distinct symbols, such that each row or column contains each symbol exactly once
  - Usually numbers, but notice that Sudoku are not really math puzzles! The numbers could be replaced with, e.g., fruits!
- Sudoku are just Latin squares with the additional  $3 \times 3$  "boxes" constraint
  - There are  
5524751496156892842531225600  
9x9 Latin squares,  
6670903752021072936960  
(0.0001%) of which are Sudoku (you see why newspapers don't run out...)

1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	1
3	4	5	6	7	8	9	1	2
4	5	6	7	8	9	1	2	3
5	6	7	8	9	1	2	3	4
6	7	8	9	1	2	3	4	5
7	8	9	1	2	3	4	5	6
8	9	1	2	3	4	5	6	7
9	1	2	3	4	5	6	7	8

*swapping pairs of rows/columns retains the property!*

# "Greco-Latin" squares

- Like Latin squares but with two sets of symbols; also, each cell has a *different pair* of symbols.
  - Also called mutually orthogonal

A $\alpha$	B $\delta$	C $\beta$	D $\epsilon$	E $\gamma$
B $\beta$	C $\epsilon$	D $\gamma$	E $\alpha$	A $\delta$
C $\gamma$	D $\alpha$	E $\delta$	A $\beta$	B $\epsilon$
D $\delta$	E $\beta$	A $\epsilon$	B $\gamma$	C $\alpha$
E $\epsilon$	A $\gamma$	B $\alpha$	C $\delta$	D $\beta$

# "Greco-Latin" squares

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A very curious question, which has exercised for some time the ingenuity of many people, has involved me in the following studies, which seem to open a new field of analysis, in particular the study of combinations. The question revolves around arranging 36 officers to be drawn from 6 different regiments so that they are ranged in a square so that in each line (both horizontal and vertical) there are 6 officers of different ranks and different regiments.

— Leonhard Euler

Euler conjectured that no such 6 x 6, 10 x 10, 14 x 14 etc. Greco-Latin squares existed.

- This is a rare instance of Euler being wrong ("Euler spoilers" were found) – it turns out that *only* 6 x 6 are impossible!

Aα	Bδ	Cβ	Dε	Eγ
Bβ	Cε	Dγ	Eα	Aδ
Cγ	Dα	Eδ	Aβ	Bε
Dδ	Eβ	Aε	Bγ	Cα
Eε	Aγ	Bα	Cδ	Dβ



# Latin squares in experimental design

- You are testing out a new kind of plant food. You want to test what happens if you water plants with 0%, 25%, ..., 100% solutions of it.
- You set up a 5x5 grid of identical flowerpots on a table, and administer the treatments like this:

0%	25%	50%	75%	100%
0%	25%	50%	75%	100%
0%	25%	50%	75%	100%
0%	25%	50%	75%	100%
0%	25%	50%	75%	100%

- You find that the plants that got higher concentrations of the food grow taller! You excitedly publish and then your paper gets rejected. **Why?**

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- You set up a 5x5 grid of identical flowerpots on a table, and administer the treatments like this:

0%	25%	50%	75%	100%
25%	50%	75%	100%	0%
50%	75%	100%	0%	25%
75%	100%	0%	25%	50%
100%	0%	25%	50%	75%

*Better! Now any unexpected left-to-right factor (e.g., light levels?) is kinda controlled for*

- You find that the plants that got higher concentrations of the food grow taller! You excitedly publish and then your paper gets rejected. Why?
  - Use a Latin square design instead! (Or even Greco-Latin for 2 treatments)



# Latin squares in experimental design

- You are testing out a new kind of plant food. You want to test what happens if you water plants with 0%, 25%, ..., 100% solutions of it.
- You set up a 5x5 grid of identical flowerpots on a table, and administer the treatments like this:

galaxy brain: maybe a Sudoku is actually even better! since it doesn't have these diagonal lines...

Can you think of why it might not be better?

0%	25%	50%	75%	100%
25%	50%	75%	100%	0%
50%	75%	100%	0%	25%
75%	100%	0%	25%	50%
100%	0%	25%	50%	75%

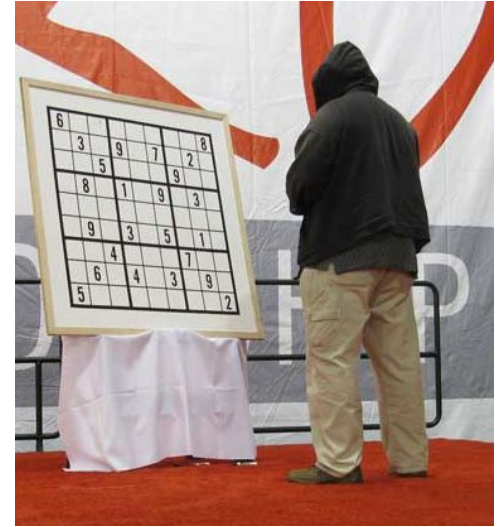
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# Interlude: Cheating at Sudoku

At the chess tournament, "Varshavsky" defeated a far higher-rated opponent with moves that matched those that a computer program would have suggested. Before he could be searched, he disappeared into a bathroom stall for about 10 minutes.

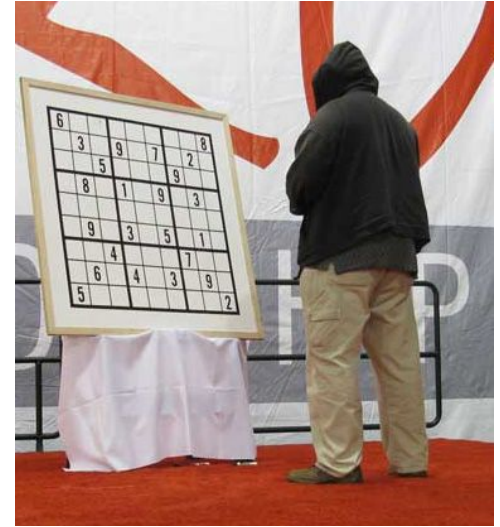
At this past weekend's sudoku tournament, "Varshavsky" "blazed through the second round in world-class time," the *Inquirer* says, but then couldn't figure out the "easy first steps in the championship puzzle." He's described in the *Inquirer*, by the way, only as "playing in a hooded sweatshirt."



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**“The reexamination results were very much consistent with Mr. Varshavsky’s onstage performance,”** said Will Shortz, *The New York Times’s* puzzle editor and a tournament organizer, in a statement. **“We have concluded that Eugene Varshavsky alone could not have solved the Round Three puzzles during the championship...”**

# Constructing Sudoku

- Three parts to logic puzzle construction:
  - Have a way of generating candidate puzzles
  - Verify that a candidate puzzle has a unique solution (not necessarily a unique solution path)
  - Make sure the puzzle is actually satisfying and fun for humans to solve **(this is the truly hard part, in my experience)**
- All this means that the human/computer constructor also really has to understand how to solve the puzzles, too.

# Solving Sudoku

- Try a bunch of increasingly complicated algorithms
- Resort to brute force exhaustive guess-and-check if all else fails
  - And we can't just claim this happens rarely...

Candidates can be Edited Highlighted Chained / Shown

1	3	1	1	7	2	4	1	3	3	5
8	9	8	6	8	6	8	8	6	8	8
A										
B										
C										
D										
E										
F										
G										
H										
J										

Version 2.09  
See Strategy Overview documentation

**X-Wing**  
(Row->Col) 4 taken off C3, based on CE28  
(Row->Col) 4 taken off C7, based on CE28  
(Row->Col) 4 taken off E7, based on CE28

**Take Step** <<

Check for solved cells 0  
Show Possibles No  
1: Hidden Singles No  
2: Naked Pairs/Triples No  
3: Hidden Pairs/Triples No  
4: Naked/Hidden Quads No  
5: Pointing Pairs No  
6: Box/Line Reduction No  
**Tough Strategies**  
7: X-Wing Yes  
8: Simple Colouring  
9: Y-Wing  
10: Swordfish  
11: XYZ Wing  
12: BUG  
**Diabolical Strategies**  
13: X-Cycles  
14: XY-Chain  
15: 3D Medusa  
16: Jellyfish  
17: Unique Rectangles  
18: Fireworks  
19: SK Loops  
20: Extended Unique Rect.  
21: Hidden Unique Rect's  
22: WXYZ Wing  
23: Aligned Pair Exclusion  
**Extreme Strategies**  
24: Exocet  
25: Grouped X-Cycles  
26: Empty Rectangles  
27: Finned X-Wing  
28: Finned Swordfish  
29: Altern. Inference Chains  
30: Sue-de-Coq  
31: Digit Forcing Chains  
32: Nishio Forcing Chains  
33: Cell Forcing Chains  
34: Unit Forcing Chains  
35: Almost Locked Sets  
36: Death Blossom  
37: Pattern Overlay Method  
38: Quad Forcing Chains  
"Trial and Error"  
39: Bowman's Bingo

Key: Strong Link Group/ALS

# Sudoku is NP-complete (probably intractable)

- NP-complete means:
  - it is quick (takes polynomial time) to check a solution once you have it (this is the "NP" part)
    - **Nondeterministic** polynomial time = you have a machine that guesses every solution path at once
  - but if we had a way of finding a solution, we could use it (in polynomial time) to solve any other problem that can be checked in polynomial time (i.e. any other "NP" problem)

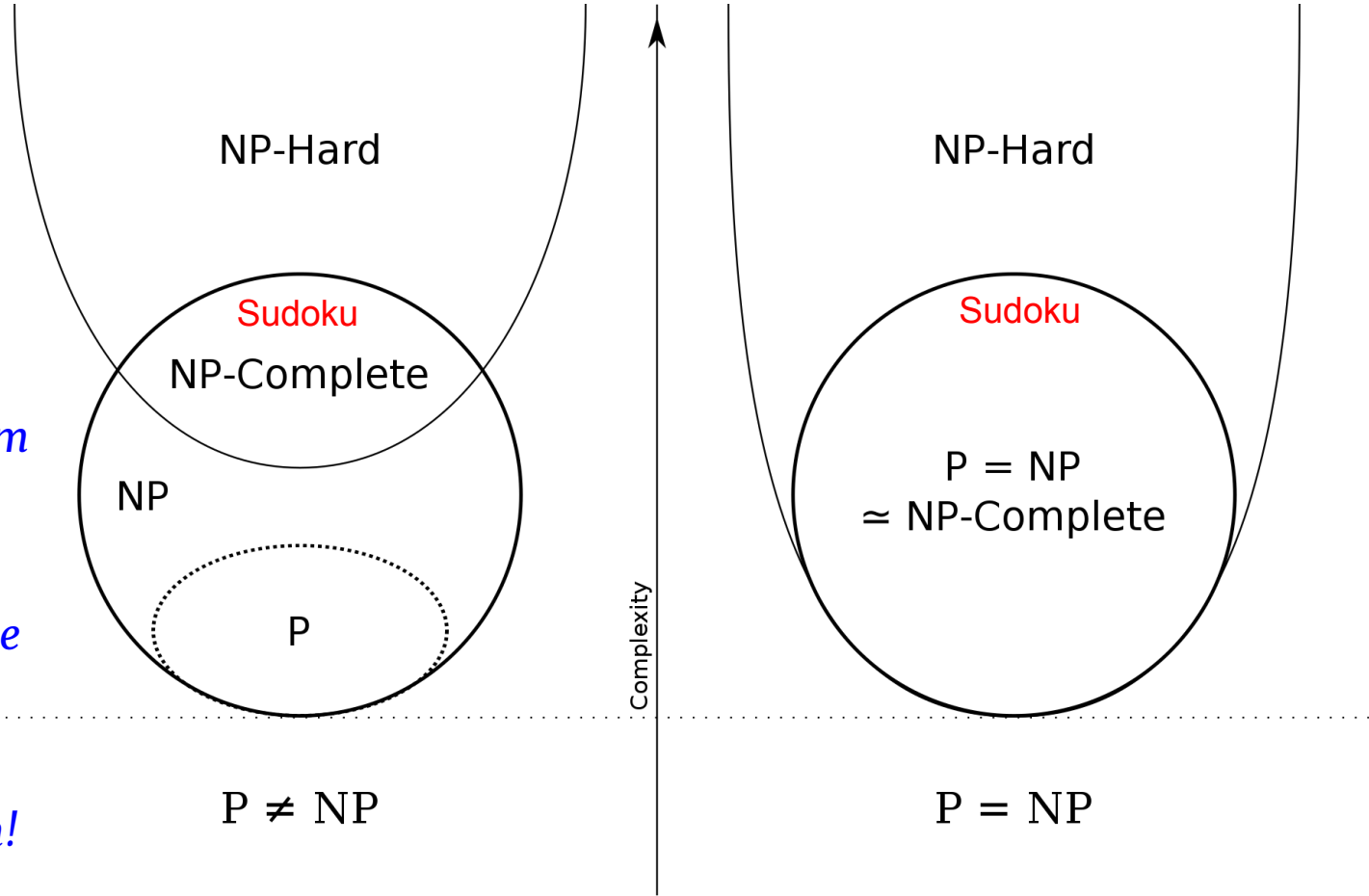
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  - but if we had a way of finding a solution, we could use it (in polynomial time) to solve any other problem that can be checked in polynomial time (i.e. any other "NP" problem)
- 3SAT can be reduced to (i.e., reframed in terms of) a problem called Tripartite Graph Triangulation...
  - which can be reduced to a problem called Latin Square Completion...
    - which can be reduced to Sudoku
- Are Sudoku and 3SAT in the complexity class P (i.e., solvable in polynomial time?)

# Well, which is it?

*Solve this and*

- *win \$1 million!  
(Clay Millennium Prize)*
- *win the awe of  
the math and  
computer science  
communities!*
- *possibly destroy  
the world  
financial system!*





# A common misconception



Mark Gritter · [Follow](#)



PhD dropout in Computer Science, Stanford University (Graduated 2006) · Upvoted by Justin Rising, [MSE in CS](#) and Timothy Johnson, [PhD Computer Science, University of California, Irvine \(2018\)](#) · Updated 1y

**Related** **How fast to solve Sudoku 9x9 to prove P=NP? My code solves them in 1/5th of a second with 20 hints and the toughest known 9x9 in 2.3 seconds.**

You cannot prove  $P=NP$  on a 9x9 Sudoku. You will need, at the very least, to demonstrate that your algorithm continues to perform well for larger instances. 9x9 Sudoku is not NP-complete; *generalized*  $N^2 \times N^2$  Sudoku is NP-complete.

# Both of these are true:

- Solving 9x9 Sudoku is relatively "easy" for computers, in practice, but we are using exponential (non-polynomial) algorithms.
- Solving *arbitrarily* large Sudoku (e.g., 100x100 with 10x10 blocks) is thought to be computationally intractable (no polynomial-time algorithm exists)
  - if it did, it would imply that a lot of other problems we think are intractable are also tractable

**Takeaway:** Although pretty much all Nikoli puzzles turn out to be NP-complete, that doesn't mean we can't solve and construct fun human-sized ones!



# Construction consideration: Uniqueness

- Most solvers expect a puzzle to have a unique solution.
  - It's less satisfying otherwise?
- This Sudoku with 77 of 81 numbers filled in does not have a unique solution. Why not?

1	4	5	3	2	7	6	9	8
8	3	9	6	5	4	1	2	7
6	7	2	9	1	8	5	4	3
4	9	6		8	5	3	7	
2	1	8	4	7	3	9	5	6
7	5	3		9	6	4	8	
3	6	7	5	4	2	8	1	9
9	8	4	7	6	1	2	3	5
5	2	1	8	3	9	7	6	4

# Construction consideration: Uniqueness

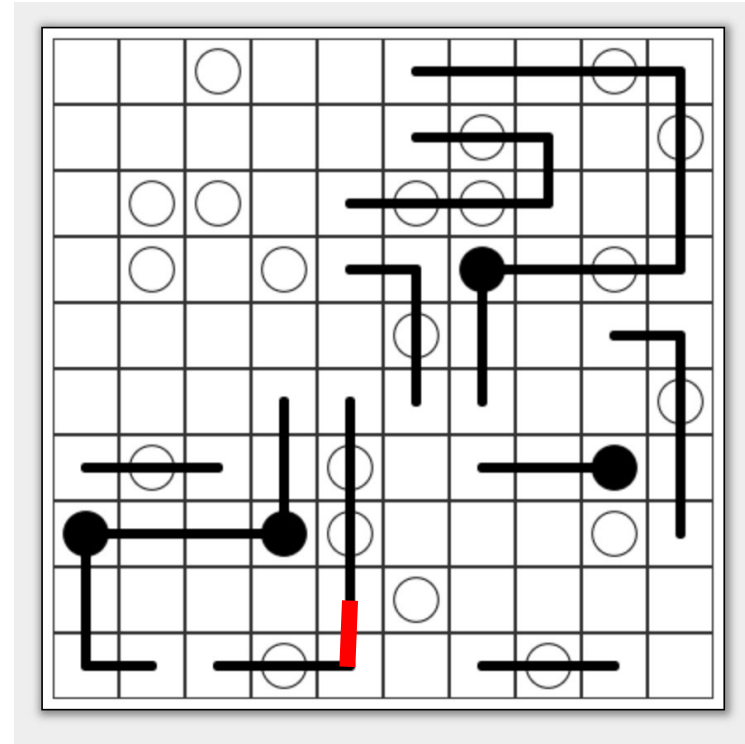
- Most solvers expect a puzzle to have a unique solution.
  - It's less satisfying otherwise?
- This Sudoku with 77 of 81 numbers filled in does not have a unique solution. Why not?
  - If you try to construct any kind of Nikoli-like puzzle, you will *almost certainly* run into this right away.
  - Worse yet are the versions of this that are harder to spot, involving long loops

1	4	5	3	2	7	6	9	8
8	3	9	6	5	4	1	2	7
6	7	2	9	1	8	5	4	3
4	9	6		8	5	3	7	
2	1	8	4	7	3	9	5	6
7	5	3		9	6	4	8	
3	6	7	5	4	2	8	1	9
9	8	4	7	6	1	2	3	5
5	2	1	8	3	9	7	6	4



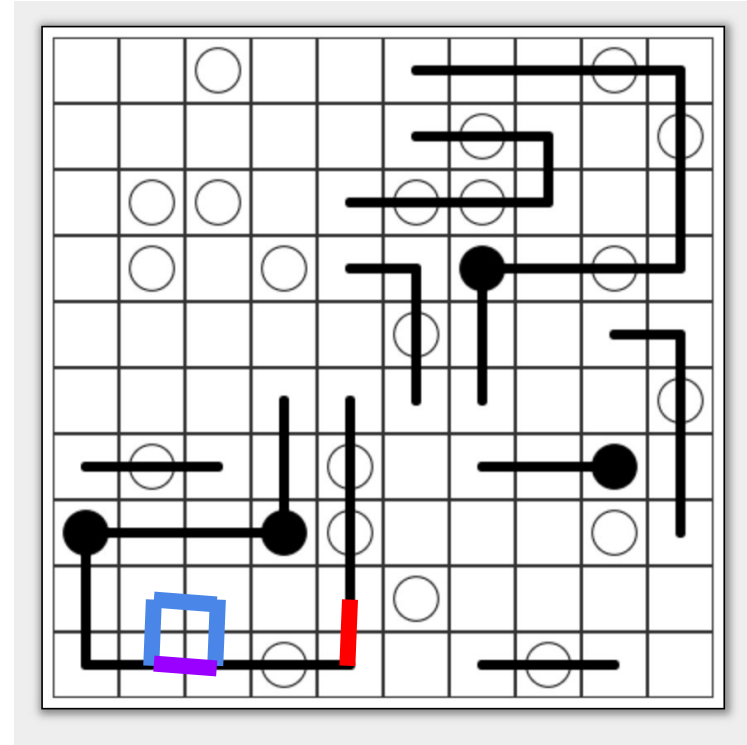
# Uniqueness interlude

- Masyu ("Evil Influence"): You have a grid with some white and black pearls.
  - Draw a single closed loop that goes through every pearl once and doesn't touch itself.
  - The path must go straight through white pearls, but must bend directly immediately before or after (or both)
  - The path must bend in black pearls, but must go straight in the cells immediately before and after
- Suppose we added this red line.



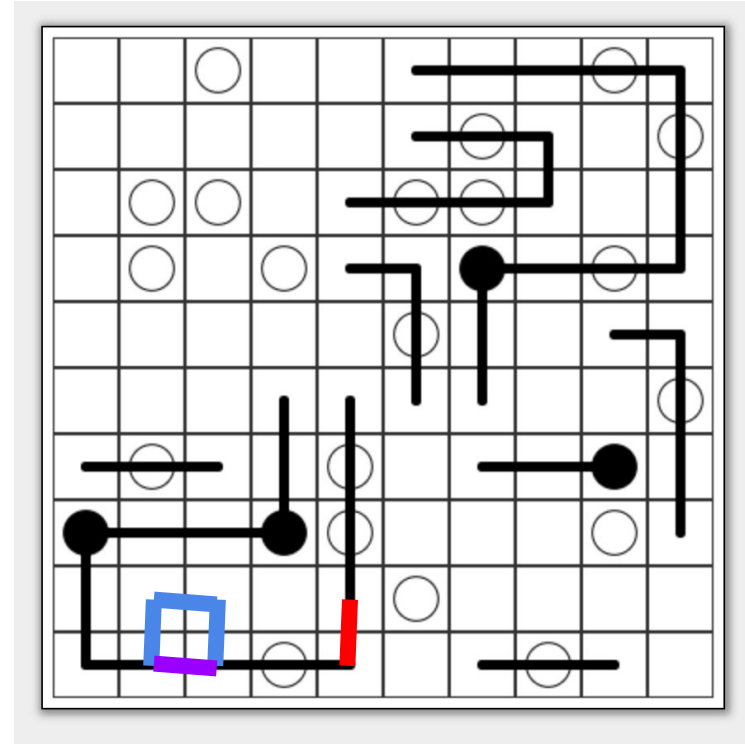
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  - But now there are **two solutions!**
  - So we know we can't add the red line.



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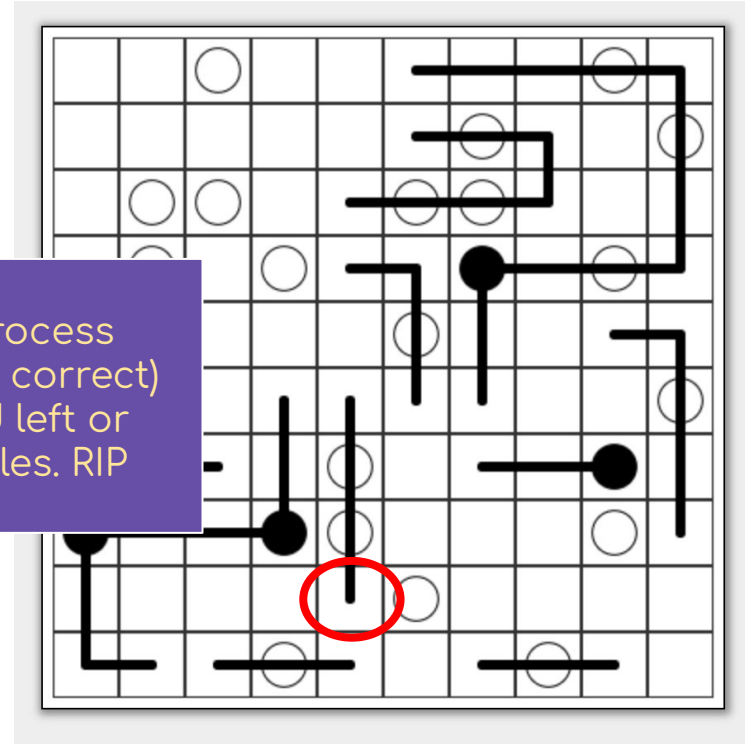
*Convention: Solvers can use uniqueness but should never be forced to.*



# Uniqueness interlude

- Masyu ("Evil Influence"): You have a grid with some white and black pearls.
  - Draw a single closed path that visits every pearl once
  - The path must go straight through white pearls, but must bend before or after (or both)
  - The path must bend in black pearls, but must go straight in the cells immediately before and after
- Suppose we added this red line.
  - But now there are two solutions!
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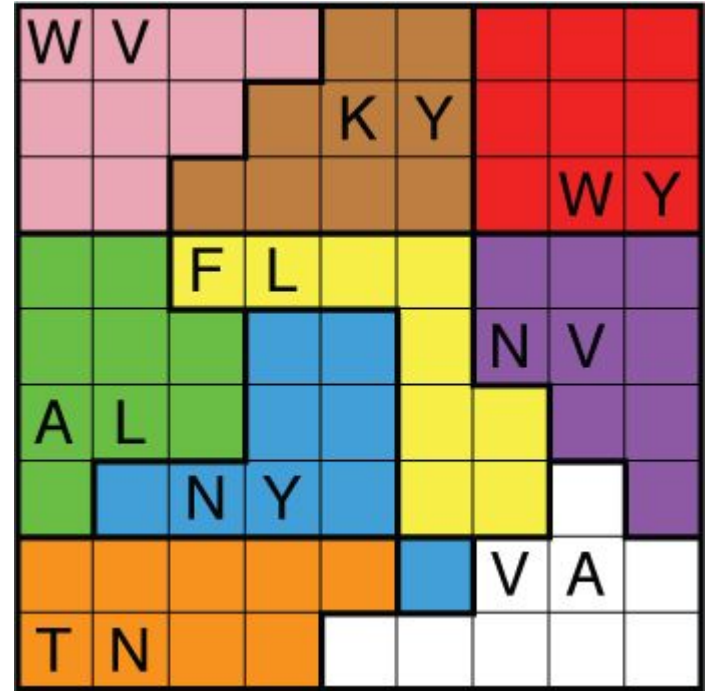
actually my whole thought process was unnecessary here (albeit correct) because the line has to bend left or right here anyway, per the rules. RIP



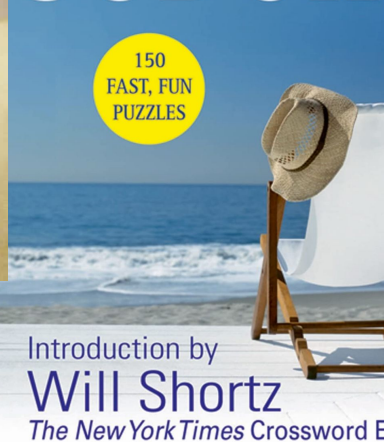
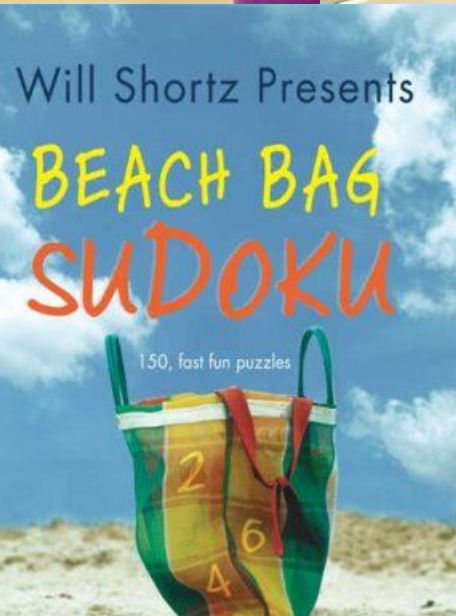
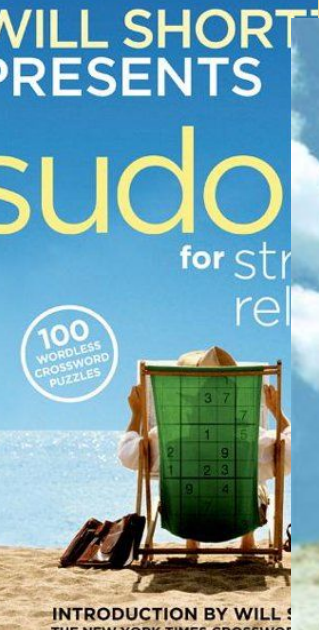
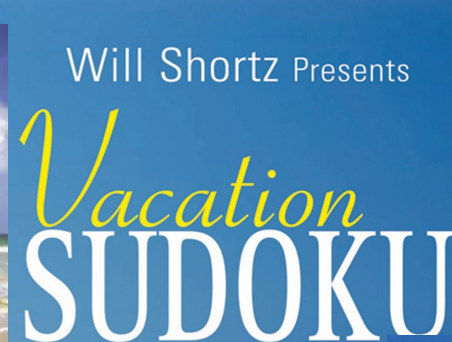
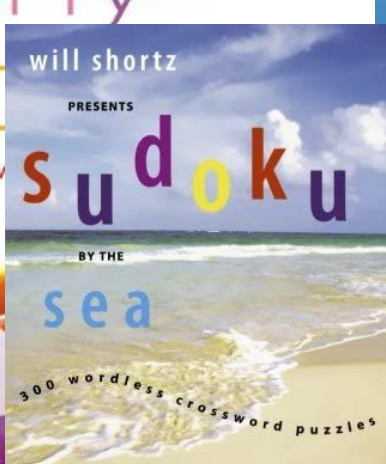
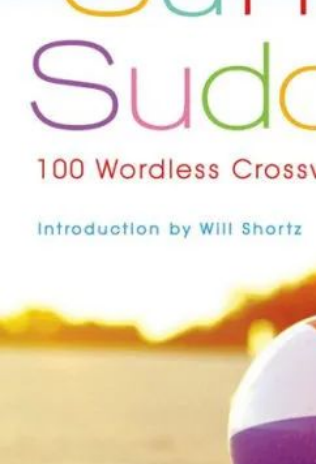
*Convention: Solvers can use uniqueness but should never be forced to.*

# Construction consideration: Fun / Beauty

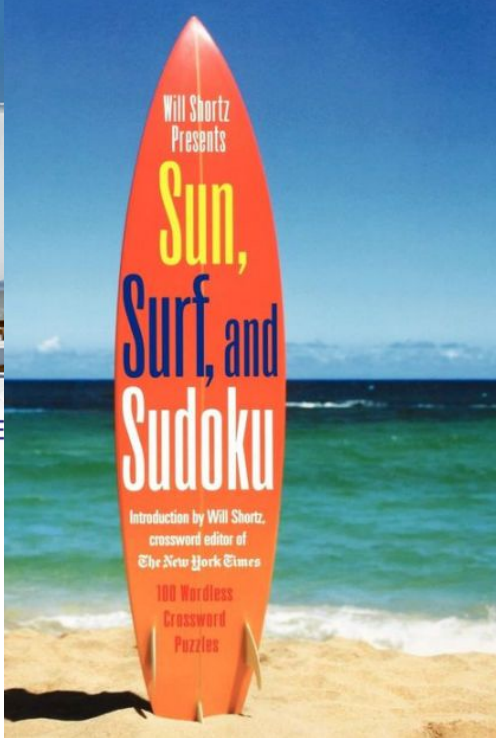
- How to ensure that a puzzle is fun for humans?
- Nikoli's philosophy: beautiful handcrafted puzzles, credit authors

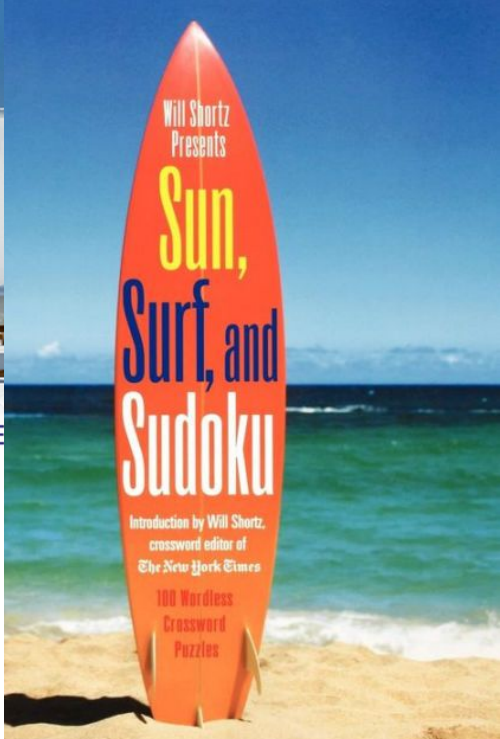
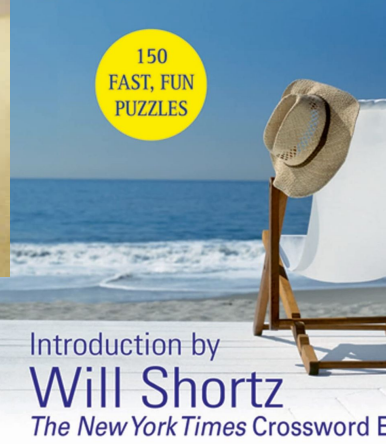
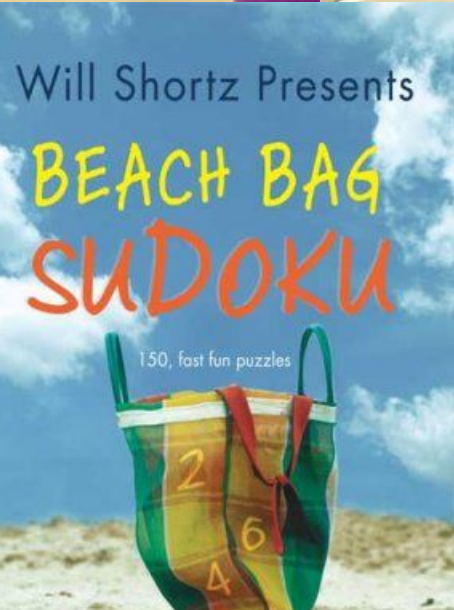
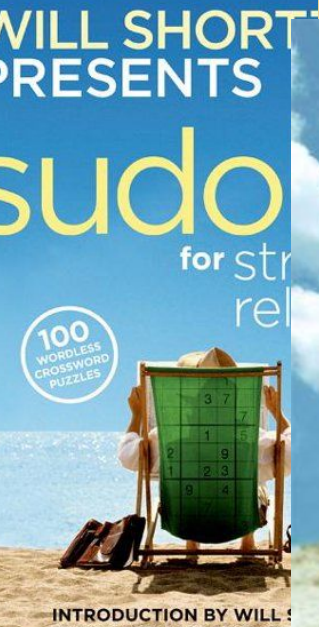
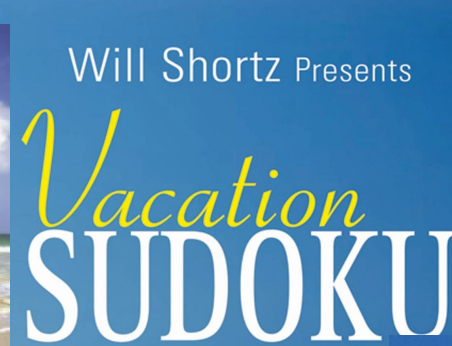
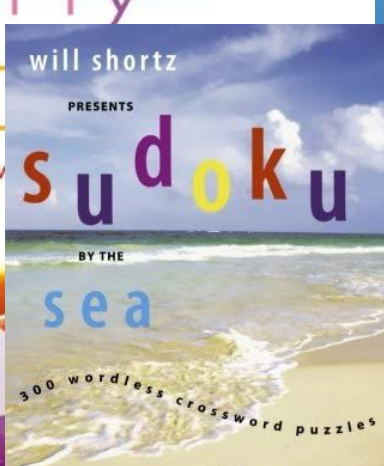
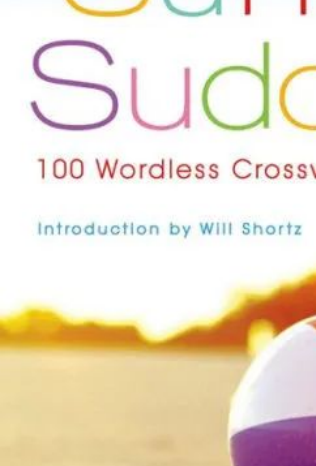


*Thomas Snyder's "State Sudoku"*



*But newspapers etc. need to create tons of Sudoku...*

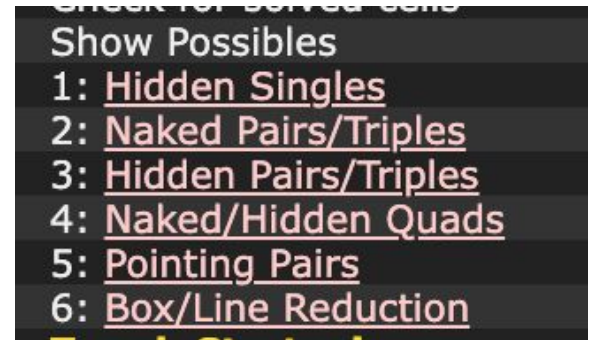




But newspapers etc. need to create tons of Sudoku...

# One quick and dirty way to do it

- Come up with a list of basic steps / heuristics that are allowable. The computer only gets to use those when checking.
- Start with a grid with relatively few "givens", add more as needed (in a brute-force, guess-and-check way) to make the puzzle solvable while still enforcing uniqueness
- It may be possible for solvers to use more advanced techniques, but who cares (besides a handful of puzzle snobs like me)



*Now you, too, can go back in time to the mid-2000s and churn out tons of Sudoku! (but bring your modern laptop)*

# Writing a different logic puzzle type

- Me: It's an outrage that the numbers in Sudokus / Latin squares don't actually mean anything! (apart from representing different symbols)
- What if we had a Latin square but where each row has exactly one 1, two 2s, three 3s... so then at least the numbers mean something...



# Writing a different logic puzzle type

- Me: It's an outrage that the numbers in Sudokus / Latin squares don't actually mean anything! (apart from representing different symbols)
- What if we had a Latin square but where each row has exactly one 1, two 2s, three 3s... so then at least the numbers mean something...
- **Good news:** These turn out to be very easy to find. (*board example*)
- **Bad news:** Latin square puzzles with missing entries are pretty much just less fun Sudoku.

# Adding constraints can be liberating

- This is a counterintuitive feature of puzzle designs! Constraining the space of what you have to think about can actually make it *easier* to be creative
- What if we had a Latin square but where each row has exactly one 1, two 2s, three 3s, and also **imposes Fillomino constraints?**



8	8	3	3	10	10	10	10	5
8	8	8	3	10	10	10	5	5
3	3	8	10	10	4	4	4	5
1	3	8	3	10	2	2	4	5
2	2	8	3	3	1	3	2	2
6	6	2	2	1	3	3	1	3
6	4	4	4	2	2	1	3	3
6	4	2	2	4	3	3	4	4
6	6	4	4	4	1	3	4	4

*Example of a Fillomino puzzle. All 3s have to be in contiguous groups of 3 that touch no other 3s, etc.*



# Problem: this does not actually work

- What if we had a Latin square but where each row has exactly one 1, two 2s, three 3s but also imposes Fillomino constraints?
- We can get really close... but an exhaustive computer search confirms that there are no such grids.

3	3	2	2	1	3
3	2	3	1	2	3
1	2	3	3	2	3
3	3	2	2	3	1
2	3	1	3	3	2
2	1	3	3	3	2

*This one is heartbreaking! It all works except for the part indicated by the red line.*

# What if we make it even harder?

- Handwavily, the problem is that the 1-regions and 2-regions are too puny to keep the big 3-regions from touching.
  - Is it Euler's 6x6 curse all over again?
- Would it actually help if we did this with a 10x10 grid with 1, 2, 3, 4- regions instead of 1, 2, 3?



3	3	2	2	1	3
3	2	3	1	2	3
1	2	3	3	2	3
3	3	2	2	3	1
2	3	1	3	3	2
2	1	3	3	3	2

- My (not optimal) method:
  - Enumerate all the possible valid rows (lists of 10 numbers with one 1, two 2s, three 3s, four 4s). There are 12600 of these.
  - Plunk down an initial row.

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    - local problems? (e.g., now there are four 3s all touching each other)
      - mildly ugly casework involving a flood-fill-like search
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  - Backtrack when an in-progress solution cannot be extended.
  - Balance breadth and depth: Try starting with all possible valid rows, spending only 10 seconds or so on each.
  - **The key CS combinatorialist technique:** Start it running, then go to sleep

# It works!

- I went from: is there even one such grid?

to: oh, there are actually quite a few of these!

to: But can we do better?



(Friday's puzzle set will have a special example!)

```
4 1 4 2 3 3 4 4 3 2
4 3 4 2 3 1 4 4 3 2
4 3 4 3 4 2 1 2 3 4
4 3 4 3 4 2 3 2 1 4
2 2 1 3 4 4 3 3 4 4
3 4 2 4 3 3 4 4 2 1
3 4 2 4 1 3 4 4 2 3
3 4 3 4 2 4 2 1 4 3
1 4 3 4 2 4 2 3 4 3
2 2 3 1 4 4 3 3 4 4
```

```
* . * . . * * . .
* . * . . * * . .
* . * . * . . . *
* . * . * . . . *
. . . * * . . * *
. * . * . * * . .
. * . * . * * . .
. * . * . * . * .
. * . * . * . * .
. . . * * . . * *
```

*boring, repetitive  
(kinda looks like a  
person flexing  
though)*

```
4 4 3 3 4 4 1 3 2 2
4 3 2 3 4 4 2 3 1 4
4 3 2 4 1 3 2 3 4 4
2 3 4 4 2 3 4 1 4 3
2 4 1 4 2 3 4 4 3 3
4 4 3 3 4 1 3 4 2 2
3 4 3 2 4 4 3 2 4 1
3 1 4 2 3 4 3 2 4 4
3 2 4 1 3 2 4 4 3 4
1 2 4 4 3 2 4 4 3 3
```

```
* * . . * * . . . .
* . . . * * . . . *
* . . * . . . . * *
. . * * . . * * . .
. * . * . . * * . .
* * . . * . . * . .
. * . . * * . . * .
. . * . . * . . * *
. . * . . * * . *
. . * * . . * * . .
```

*better, but  
they're all in  
10x2 stripes...*

```
2 2 3 4 3 1 4 4 4 3
4 1 3 4 3 4 2 4 2 3
4 4 3 4 3 4 2 1 2 3
4 3 2 4 2 4 3 3 1 4
3 3 2 1 2 4 3 4 4 4
1 4 4 3 4 3 4 2 3 2
4 4 1 3 4 3 4 2 3 2
2 2 4 3 4 3 4 4 3 1
3 3 4 2 4 2 1 3 4 4
3 4 4 2 1 2 3 3 4 4
```

```
. . . * . . * * * .
* . . * . * * . .
* * . * . * . . . .
* . . * . * . . . *
. . . . * * * *
. * * . * . * . .
* * . . * . * . .
. . * . * . * * . .
. . * . * . . * *
. * * . . . . * *
```

*this is more  
interesting?*

# Still to come in part 2...

- More Nikoli puzzles (e.g., Kakuro, which feels different from the others and is actually a math puzzle)
- If we're stuck doing exponential searches, can we at least make them as fast as possible?

